# Improvement of the IVS-INT01 Sessions through Bayesian Estimation

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**Abstract** We report on the use of Bayesian estimation in the analysis of IVS INT01 sessions. We demonstrate that the use of a priori knowledge improves the accuracy of the UT1 estimates. We look at two cases—the incorporation of gradients estimated from independent R1 and R4 sessions and the use of an external model of Free Core nutation. In each case we find the magnitude of the change in the UT1 estimates induced by using the a priori information. We also calculate the improvement in the accuracy of the Intensives as measured by the agreement between the Intensive estimate of UT1 and that of a concurrently run R1/R4 session. In both cases, the accuracy is improved, and the amount of improvement is consistent with expectations based on the size of the effect.

**Keywords** Intensive, UT1, Bayesian estimation

## 1 Introduction

VLBI makes important contributions to the estimation of Earth Orientation Parameters and is unique in its ability to measure UT1 [1]. Because of this, the IVS schedules bi-weekly 24-hour R1 (Monday) and R4 (Thursday) sessions to measure all components of EOP. The IVS also schedules special one-hour sessions designed specifically to measure UT1. These Intensive sessions have a small number of stations (typically two to four) involving long East-West baselines and run for about one hour, resulting in 15–40 observations. Normally data from these sessions is transmitted electronically to the correlator, and the final UT1 estimate is often available within $<$ 24 hours after the session ends. But the very characteristics of short duration and small network size that makes rapid transmission, correlation, and analysis possible also means that the precision of the Intensives is much less than that of 24-hour VLBI sessions. Anything that can improve the accuracy and/or the precision of the Intensives is important.

In this paper our data set is the INT01 series which are scheduled by USNO, run Monday–Friday, and use the Kokee–Wettzell baseline. These sessions are scheduled using two alternating strategies. The STN uses a small set of strong sources with uneven sky coverage. The MSS uses a large set of sources that are on average weaker but have good sky coverage. Sessions can behave differently depending on the strategy used to schedule them, so we typically divide the INT01 sessions into STN and MSS subsets for analysis [2]. Ultimately we are interested in the accuracy of the UT1 estimates. Because of this we further restrict our attention to only those Intensives that occur on the same day as an independent R1/R4 session. Our proxy for the accuracy is the difference between the Intensive UT1 estimate and that of the concurrent R1/R4 session interpolated to the same epoch. Figure 1 plots this difference for 2011–2012. The STN estimates differ from the 24-hour estimates by up to $\sim$100 µs, and the MSS estimates differ by up to $\sim$70 µs. The standard deviations of the differences are also high—30.68 µs for the STN and 21.04 µs for the MSS.

Our primary goal is to improve the a priori models used in the Intensives, and to verify that this results in more accurate UT1 estimates. As a side-effect, we determine the change in the UT1 estimates caused by changing the a priori. This is similar to work done previously by Nothnagel and Schnell [5] who looked at

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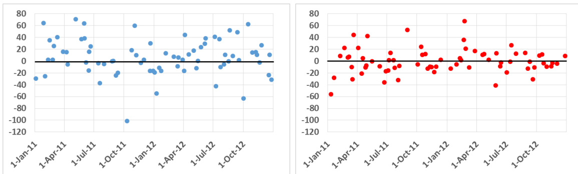


**Fig. 1** Differences between UT1 estimates from IVS 24-hour and operational INT01 STN (left) and MSS (right) sessions.

the effect of errors in Polar Motion (PM) and Nutation (FCN) and found they were signficant. Gipson et al. [3] redid this work and extended it to include the effect of errors in station position, mapping functions, and atmosphere gradients. They determined which modeling errors have potentially the largest impact on UT1 (East, PM, FCN, gradients) and hence should have a priority on improved modeling, and which error sources had a small effect (e.g., Up and North) and could be ignored.

Gipson et al. [3] also demonstrated that using PM and nutation estimates derived from concurrent 24-hour R1/R4 sessions in Intensive analysis improved the agreement between the Intensive and R1/R4 estimates of UT1. We extend this work by demonstrating that using atmospheric gradients from 24-hour sessions also improves the agreement. Such an approach is not possible operationally, because the results from the R1/R4 sessions are not available until several weeks later. We also looked at improving the a priori modeling by using data which *is* available at the time of the Intensive processing, namely using FCN values from Sébastien Lambert's FCN series [4]. We find that use of this data improves the agreement between the Intensive and 24-hour session estimates.

In Sections 2 and 3 we discuss our general approach. In Section 4 we look at using gradient estimates from R1/R4 sessions. In Section 5 we look at applying external FCN. Section 6 has some concluding remarks. We find that the use of better a priori information improves the accuracy of Intensive UT1 estimates and that the magnitude is in line with what you would expect based on the effect of the error. But the resulting change in the accuracy is small, 1–2 µs or less.

## 2 Augmented Normal Equations

Least squares estimation involves solving the matrix equation $A = N^{-1}B$, where A is the parameter estimate vector, $N$ is the normal matrix of weighted partial derivatives, and $B$ is the '$O$–$C$' (="Observed minus Calculated") vector. INT01 sessions have a small ($\sim$ 16 to 25) number of observations and only allow estimation of a few (typically five) parameters which are: atmosphere offsets at Kokee and Wettzell; clock and clock rate at Wettzell; and UT1 offset. Sometimes a quadratic clock term is estimated as well.

An important and often unstated implicit assumption is that the a priori models are correct. We know that this assumption is incorrect, because, for example, when we estimate PM in a 24-hour session we get non-zero values. An error in an underlying model will change the 'O–C' vector, which in turn will change the estimates.

To address this issue, we construct the normal equations for the Intensives with additional parameters corresponding to possible errors in the a priori. We call these 'augmented' normal equations. These may be singular because we may not have enough data to estimate the parameters. To fix this we apply additional information in the form of constraints. We call this approach 'Bayesian'. Explicitly, let $a$ be the index of an extra parameter $A_a$, and assume that we know that $A_a$ has the value $V_a$ with an uncertainty of $\sigma_a$. We modify the normal equations thus: $N_{aa} \rightarrow N_{aa} + 1/\sigma_a^2$, $B_a \rightarrow B_a + V_a/\sigma_a^2$ with all other components staying the same. Effectively we are introducing extra "observations" corresponding to the constraint. If $V_a = 0$, in the limit $\sigma_a \rightarrow 0$ we recover the usual UT1 estimate. Non-zero values of $V_a$ and $\sigma_a$ change the UT1 estimate.

## 3 Effect of Changing a Priori

In this section we establish some notation and derive some results. Let $j$ label an Intensive, and let $UT1_{Def,j}$ (respectively, $UT1_{Mod,j}$) be the UT1 estimate from the default (respectively, modified) processing. Also let $UT1_{24,j}$ be the UT1 estimate from a corresponding 24-hour session, interpolated to the epoch of the Intensive. We are interested in the following quantities:

$$\Delta UT1_{Def,j} = UT1_{Def,j} - UT1_{24,j} \quad (1)$$
$$\Delta UT1_{Mod,j} = UT1_{Mod,j} - UT1_{24,j} \quad (2)$$
$$\delta UT1_{Mod,j} = UT1_{Def,j} - UT1_{Mod,j} \quad (3)$$

The first two are the differences between the Intensive and the 24-hour estimate of UT1, and will be useful in determining the accuracy of Intensive UT1 estimates. The third defines the effect of modifying the analysis of the Intensive. For any series $f$, $g$, define:

$$< f \cdot g >= \sum_j^N f_j g_j / N \tag{4}$$

As $N$ goes to infinity this is just the expectation value.

Let $\varepsilon_{A,j}$ denote the error in UT1 (here $A$ is one of $Def$, $Mod$, or 24). Then we find, for example,

$$\Delta UT1_{Def,j} = \varepsilon_{Def,j} - \varepsilon_{24,j} \tag{5}$$

Consider

$$\begin{aligned} < \Delta UT1^2_{Def} > &= < \varepsilon^2_{Def} > + < \varepsilon^2_{24} > \\ & -2 < \varepsilon_{Def} \cdot \varepsilon_{24} > \end{aligned} \tag{6}$$

The errors in the 24-hour and Intensive UT1 estimates should not be correlated. Hence for large $N$ the last term on the Right Hand Side (RHS) should vanish. Further, because the formal errors for the Intensives are a factor of 5–10 larger than for the 24-hour sessions we expect that $\varepsilon^2_{Def} >> \varepsilon^2_{24}$ . This means the second term can be ignored. We are left with:

$$< \Delta UT1^2_{Def} > \simeq < \varepsilon^2_{Def} > \tag{7}$$

The symbol $\simeq$ means the equality holds for large $N$. This means that we can calculate the expected error ($\varepsilon_{Def}$) by calculating $< UT1^2_{Def} >$.

Suppose that we modify the analysis of the Intensives in some way. What is the expected error in these new estimates? Analogous to Equation 7, we find:

$$< \Delta UT1^2_{Mod} > \simeq < \varepsilon^2_{Mod} > \tag{8}$$

Note that by Equations 2 and 3 we have:

$$\begin{aligned} \Delta UT1_{Mod} &= UT1_{Mod} - UT1_{24} \\ &= UT1_{Def} - \delta UT1_{Mod} - UT1_{24} \\ &= \Delta UT1_{Def} - \delta UT1_{Mod} \end{aligned} \tag{9}$$

Hence

$$\begin{aligned} < \Delta UT1^2_{Mod} > &= < (\Delta UT1_{Def} - \delta UT1_{Mod})^2 > \\ &= < \Delta UT1^2_{Def} > + < \delta UT1^2_{Mod} > \\ & -2 < \delta UT1_{Mod} \cdot \Delta UT1_{Def} > \end{aligned} \tag{10}$$

We consider two alternatives. First, suppose that $\delta UT1_{Mod}$ is noise-like. In this case it will be uncorrelated with both $UT1_{Def}$ and $UT1_{24}$:

$$\begin{aligned} < \delta UT1_{Mod} \cdot UT1_{24} > \simeq 0 \\ < \delta UT1_{Mod} \cdot UT1_{Def} > \simeq 0 \end{aligned} \tag{11}$$

Using Equation 1 $< \delta UT1_{Mod} \cdot \Delta UT1_{Def} > \simeq 0$. Hence for a noise-like signal we have:

$$< \varepsilon^2_{Mod} > \simeq < \varepsilon^2_{Def} > + < \delta UT1^2_{Mod} > \tag{12}$$

As expected, adding noise increases the error. At the other extreme, suppose that $\varepsilon_{Mod}$ removes noise from $UT1_{Def}$. In this case we still expect it to be uncorrelated with $UT1_{24}$, but it should be correlated with $UT1_{Def}$:

$$\begin{aligned} < \delta UT1_{Mod} \cdot UT1_{24} > &\simeq 0 \\ < \delta UT1_{Mod} \cdot UT1_{Def} > &\simeq < \delta UT1^2_{Mod} > \\ & +2 < \delta UT1_{Mod} \cdot \Delta UT1_{Mod} > \end{aligned} \tag{13}$$

where the last term is effectively zero. Going through the same analysis of Equation 10 as previously:

$$< \varepsilon^2_{Mod} > \simeq < \varepsilon^2_{Def} > - < \delta UT1^2_{Mod} > \tag{14}$$

hence adding the signal $\delta UT1_{Mod}$ reduces the error.

## 4 Use of R1/R4 Gradients

In the analysis of the 24-hour and Intensive sessions the *a priori* gradient is the average gradient at a site computed from a numerical weather model. In the 24-hour sessions we estimate residual East-West and North-South gradients as a Piece-Wise-Linear function with rate breaks every six hours. In the Intensives we do not.

To incorporate additional information about gradients into the Intensives we augment the Intensive normal equations to include residual gradients. We restrict attention to only those Intensives that occur on the same day as R1/R4 sessions that include both Kokee and Wettzell. Out of the original 74 STN and 70 MSS sessions we were left with 54 and 53 sessions, respectively. The R1/R4 residual gradient estimates are interpolated to the epoch of the Intensives. The Intensive gradient estimates are constrained to these values with small sigmas.

In analogy with Equation 1, define the following:

$$\begin{aligned} \delta UT1_{Grad,j} &= UT1_{Def,j} - UT1Grad,j \\ \Delta UT1_{Def,j} &= UT1_{Def,j} - UT1_{24,j} \\ \Delta UT1_{Grad,j} &= UT1_{Grad,j} - UT1_{24,j} \end{aligned} \tag{15}$$

The first of these is just the change in UT1 estimates caused by including gradients and is plotted in Figure 2. The second+third items are the distance between the Intensive and 24-hour estimates. Because the errors in the 24-hour sessions are much smaller, these are a measure of the error in the Intensive estimates. Figure 3 plots $|\Delta UT1_{Def,j}| - |\Delta UT1_{Grad,j}|$ which we call the reduction in absolute error. The first term is the distance between the default Intensive and 24-hour estimate of UT1, while the second is the distance when we use gradients. If this is positive, using gradients helped.

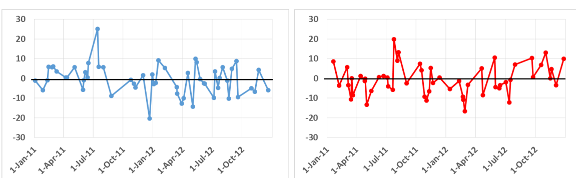

**Fig. 2** Effect of using gradient estimates on STN (left) and MSS (right) INT01 UT1 estimates (µs).

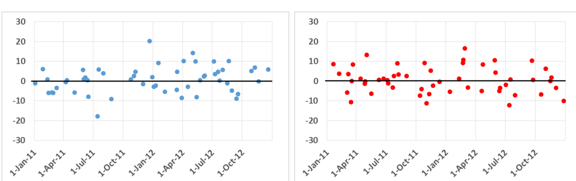

**Fig. 3** Reduction in absolute error for STN (left) and MSS (right) INT01 sessions (µs) from using gradients.

As demonstrated in the previous section, the RMS of $\Delta UT1_{Def}$ ($\Delta UT1_{Grad}$) is a measure of the error in the default (gradient) estimate of UT1 from the Intensives. Table 1 lists these values. Using gradients improves the accuracy, although the amount is small. This table also lists the RMS of $\delta UT1_{Grad}$ which is a measure of the size of the 'gradient signal' in the UT1 estimate. Gradients change the UT1 estimate by $\sim 7.5$ µs, which is a large effect. This may seem in contradiction to the small change in error. But as shown in the previous section,

$$RMS\delta UT1_{Grad} \simeq \sqrt{< \Delta UT1^2_{Def} > - < \Delta UT1^2_{Grad} >}$$

which relates the size of the gradient signal to a reduction in the variance. Table 1 displays the RHS of this equation. Note that these values are within 20% of $\delta UT1_{Grad}$ which is a resonable agreement.

**Table 1** Effect of atmospheric gradients in µs.

| | STN | MSS |
|---|---|---|
| RMS $\Delta UT1_{Def}$ | 29.30 | 21.73 |
| RMS $\Delta UT1_{Grad}$ | 28.42 | 19.84 |
| **Improvement in RMS** | **0.88** | **1.89** |
| RMS $\delta UT1_{Grad}$ | 7.46 | 7.62 |
| $\sqrt{< \Delta UT1^2_{Def} > - < \Delta UT1^2_{Grad} >}$ | **6.55** | **8.98** |
| **Average reduction in absolute error** | **0.77** | **0.58** |

## 5 Use of Empirical Free Core Nutation

Sébastien Lambert (SYRTE, Observatoire de Paris) developed and maintains a model of the FCN derived from the IERS EOP 08 C04 series. The nutation values in the IERS C04 come from VLBI estimates of the FCN, and hence Lambert's model should be consistent with our estimates from the R1s and R4s. The major obstacle to using Lambert's FCN series directly is that it is given in terms of nutation X and Y, whereas our software used $\psi$ and $\varepsilon$. We wrote software to convert between these two, and Figure 4 shows good agreement between the transformed FCN values from Lambert and our FCN estimates from the R1/R4 series.

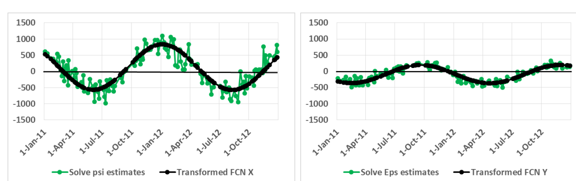


**Fig. 4** Left: agreement between transformed FCN X values and Solve $\Psi$ estimates (µas). Right: agreement between transformed FCN Y values and Solve $\varepsilon$ estimates (µas).

To use this FCN data in analyzing the Intensives we used exactly the same technique as for the gradients. We augmented the Intensive normal equations to include estimates of nutation. We interpolated the external FCN model values to the epoch of the Intensives and then applied these a priori values with tight constraints. In contrast to the previous case where we used gradient values from R1/R4 sessions, in principle we can do this for all sessions. To evaluate the effect of this, however, we limit our attention here to only those sessions which occur on the same day as an R1/R4 session.

Figure 5 plots $\delta UT1_{FCN}$, the change in UT1 estimates caused by incorporating FCN. Figure 6 plots the reduction in absolute error from using external FCN data.

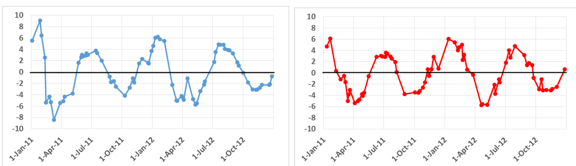

**Fig. 5** Effect of using free core nutation on STN (left) and MSS (right) INT01 UT1 estimates (μs).

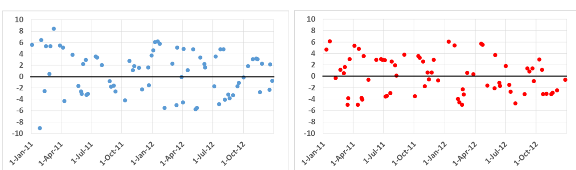

**Fig. 6** Reduction in absolute error for STN (left) and MSS (right) INT01 sessions (μs) from using FCN.

Table 2 summarizes the effect of using external FCN in the estimate of UT1 from the Intensives. The use of FCN helps, but the reduction is small, only a fraction of μs. On the other hand this reduction is in line with what you would expect based on the size of $\delta FCN$.

**Table 2** Effect of Free Core Nutation in μs.

| | STN | MSS |
|---|---|---|
| RMS $\Delta UT1_{Def}$ | 31.15 | 20.89 |
| RMS $\Delta UT1_{FCN}$ | 30.97 | 20.60 |
| **Improvement in RMS** | **0.18** | **0.29** |
| RMS $\delta UT1_{FCN}$ | 3.80 | 3.32 |
| $\sqrt{< \Delta UT1^2_{Def} > - < \Delta UT1^2_{FCN} >}$ | **3.30** | **3.49** |
| **Average reduction in absolute error** | **0.66** | **0.12** |

## 6 Conclusions

In this note we demonstrated that you can improve the accuracy of UT1 estimates from Intensive sessions by the use of a priori information. We looked at two cases—the use of gradient information from R1/R4 sessions and the use of an external FCN model. The reduction was largest when we used gradient information. But this information is not available for the operational analysis of Intensives because the R1/R4 sessions are not processed until several weeks after the Intensives are. On the other hand, there may be other sources of gradient information which are available during processing, such as from the IGS.

The use of external FCN information improved the accuracy of the UT1 estimates, but the impact was very small. This information is available at the time the Intensives are processed, and we will modify our software to use it.

We also note that in both cases, the reduction in error was larger for the MSS sessions. We believe that this is due to the fact that the error in the MSS sessions is smaller to begin with, and hence they are more sensitive to small changes in the modeling.

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